1. Refer to the problem of the two coupled oscillators discussed in Section 12.2. Show that the total energy of the system is constant. (Calculate the kinetic energy of each of the particles and the potential energy stored in each of the three springs, and sum the results.) Notice that the kinetic and potential energy terms that have $\kappa_{12}$ as a coefficient depend on $C_1$ and $\omega_1$ but not on $C_2$ or $\omega_2$. Why is such a result to be expected?

2. Find the normal coordinates for the problem discussed in Section 12.2 and in Example 12.2 if the two masses are different, $m_1 \neq m_2$. You may again assume the $\kappa$ are equal.

3. A particle of mass $m$ is attached to a rigid support by a spring with force constant $\kappa$. At equilibrium, the spring hangs vertically downward. To this mass-spring combination is attached an identical oscillator, the spring of the latter being connected to the mass of the former. Calculate the characteristic frequencies for one-dimensional vertical oscillations, and compare with the frequencies when one or the other of the particles is held fixed while the other oscillates. Describe the normal modes of motion for the system.

4. A mass $M$ moves horizontally along a smooth rail. A pendulum is hung from $M$ with a weightless rod and mass $m$ at its end. Find the eigenfrequencies and describe the normal modes.

5. A thin hoop of radius $R$ and mass $M$ oscillates in its own plane with one point of the hoop fixed. Attached to the hoop is a small mass $M$ constrained to move (in a frictionless manner) along the hoop. Consider only small oscillations, and show the eigenfrequencies are

\[ \omega_1 = \sqrt{2g/R}, \quad \omega_2 = \frac{\sqrt{2}}{2} \sqrt{\frac{g}{R}} \]

Find the two sets of initial conditions that allow the system to oscillate in its normal modes. Describe the physical situation for each mode.