1. An electric dipole \( \vec{P} \) is placed above a conducting plane, with distance \( d \) to the plane. Suppose the orientation of \( \vec{P} \) is parallel to the plane, and the plane is grounded.

(a) Use the method of images to determine the electric potential.

(b) Determine the induced charge density on the conducting plane.

(c) What is the force acting on the dipole?

**Hint.** If an electric dipole \( \vec{P} \) is placed at the origin, its electric field is given by

\[
\vec{E}(r) = \frac{1}{4\pi\varepsilon_0} \frac{1}{r^3} (3(\vec{P} \cdot \hat{r}) \hat{r} - \vec{P})
\]

2. A very long solenoid of radius \( a \), with \( n \) turns per unit length, carries a current \( I \). Coaxial
with the solenoid at radius \( b \gg a \), is a circular ring of wire, with resistance \( R \). When the current \( I_s \) in the solenoid is gradually decreased, a current \( I_r \) is induced in the ring.

(8 pt) a) Calculate \( I_r \), in terms of \( I_s = \frac{dI_s}{dt} \).

(12 pt) b) Calculate the Poynting vector just outside the solenoid.

(10 pt) * c) Show that the energy flowing out through the surface of the solenoid per unit time is just equal to the Joule power \( I_r^2 R \) dissipated in the ring due to the current \( I_r \).

(15 pt) 3. Consider a car entering a tunnel of dimension 15 m wide and 4 m high. Assume that the walls are made of good conductors, can AM radio waves propagate in the tunnel? Explain why.

Hint. The frequency of the AM radio wave is about \( 10^6 \) Hz.
4. A neutral pion $\pi^0$ moving at speed $\gamma = 0.98$ $c$ decays into two photon. If the two photons emerge on each side of the pion's flight direction with equal angles $\theta$, find the angle $\theta$ and the energies of the photons. (The rest energy of $\pi^0$ is 135 MeV.)

Hint: Express the photon energies in terms of MeV.

5. Consider a coaxial cable, consisting of a metal core with radius $a$ and a metallic shell of radius $b$ ($b > a$). The space in between is filled with a linear medium with permeability $\mu$. A current $I$ flows along the core (being distributed uniformly on the cross section), and returns on the outer shell in the opposite direction. (The permeability of the core is $\mu_0$.)

(a) Determine the $B$-field in space.
(b) Determine the bound surface current density.
(c) Determine the self-inductance of the cable per unit length.
(lopt) * d) Can you determine the capacitance (per unit length) of the cable, if the linear medium has permittivity $\varepsilon$.

Appendix

Figures for the Problems.

Problem 1.

Problem 4.

Problem 2.

Problem 5. Cross Section

$\pi^0$