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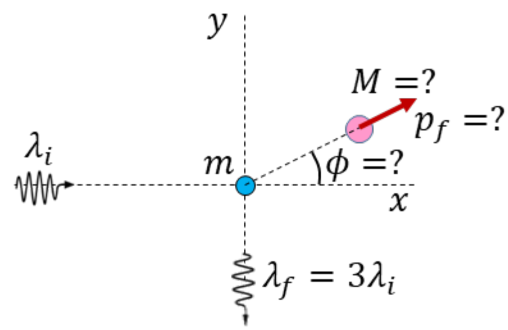
Problem 01



Deep Inelastic Photon scattering

In an inelastic (i.e. kinetic energy is not conserved) scattering experiment shown in the figure, an incident photon (the projectile) of wavelength λ_i travels in the $+x$ direction and scatters off a (target) proton of rest mass m, which is stationary. The photon scatters in the $-y$ direction with final wavelength $\lambda_f = 3\lambda_i$, and an excited baryon (a hyperon) of rest mass M is produced which recoils as shown. This is the picture in the laboratory frame K.

(a) Write down the momentum (column) 4-vectors P_1^α, P_2^α of the projectile and the target, respectively, as seen in the lab (K) frame

(b) A frame K' of reference moves at speed βc in the $+x$ direction relative to K, find the momentum 4-vectors P'^α_1, P'^α_2 of the projectile and target, respectively, as seen in the K' frame.

(c) It turns out in this case that K' is the center-of-mass frame for the original projectile and target (i.e. in K' the total 3-momentum of the two add to zero). Find β in terms of the given constants.

In these experiments, by measuring the angle and the energy and momentum of the scattering photon, you can learn everything about the recoiling product particle.

In no particular order, find (in terms of m, λ_i and constants):

(d) the recoil angle ϕ of the product hyperon in degrees,

(e) the momentum p_f of the hyperon, and

(f) the mass M of the hyperon.

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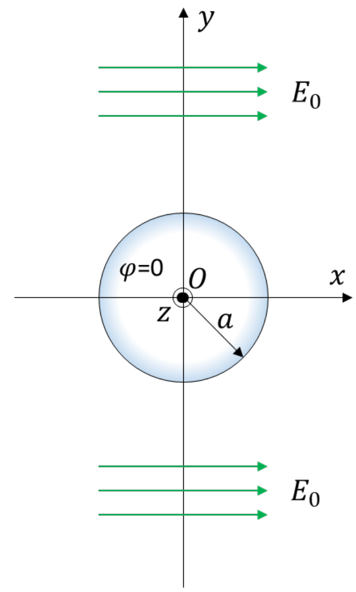
Problem 02



A solid, grounded, (i.e. its potential is fixed at c=0) infinite conducting cylinder has radius a, and is centered on the z −axis. It is placed in a space that is otherwise filled with a uniform electric field $\vec{E} = E_0\hat{x}$. In other words, very far away from the cylinder ($s \to \infty$) the electrostatic potential φ approaches that which gives $\vec{E} \to E_0\hat{x}$. The entire system is invariant with respect translations along the z −axis (i.e. the potential $\varphi(s, \phi)$ has no dependence on z.

Here s is the cylindrical radius (perpendicular distance from the z −axis, and ϕ is the azimuthal angle measured counter-clock-wise from the positive x −axis.

(a) Write down the most general form for the solution $\varphi(s, \phi)$ to the Laplace equation $\nabla^2\varphi = 0$ that is independent of z, following the guidelines below.

Your answer should be consistent with the implied periodic boundary condition $\varphi(s, \phi \pm 2m\pi) \equiv \varphi(s, \phi), m = 1,2,3$ ....

We stipulate an additional condition that $\varphi \to 0$ as $y \to \pm\infty$, at $x = 0$, such that the usual $m = 0$ terms have zero coefficients and can be neglected here. So your answer should also be an infinite series in index m, starting from $m = 1$.

Each $m \geq 1$ term in the series should have four coefficient, where (1) A_m multiply functions of s that diverges as $s \to \infty$, (2) B_m multiply functions of s that vanishes as $s \to \infty$, (3) C_m multiply even functions of ϕ, and (4) D_m multiply odd function of ϕ. Note only three of the four of these coefficients are actually independent.

(b) Apply the boundary condition of the grounded, solid, conducting cylindrical surface at $s = a$. Using the orthogonality conditions of sine and cosine functions, eliminate some of the coefficients in terms of the others.

(c) Now apply the boundary condition at $s \to \infty$ (keeping the non-vanishing asymptotic part of the series) as stated at the start of the problem and solve for $\varphi(s, \phi)$. Hint: your solution should no longer be an infinite series.

(d) Find the electric field everywhere outside the cylinder in cylindrical coordinates and cylindrical components.

(e) With the solution in (d), show that in the limit $s \to \infty$ the electric field $\vec{E} \to E_0\hat{x}$.

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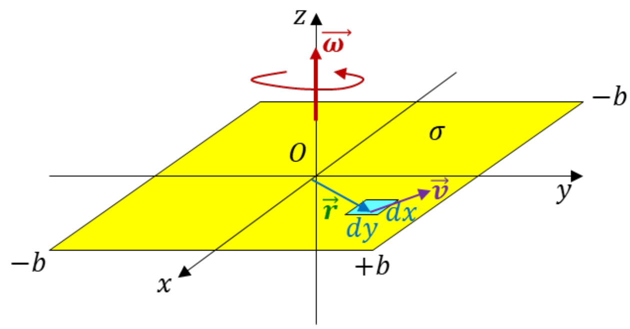
Problem 03:



A thin square plate of sides $2b$ as shown lies in the x-y plane. It is embedded with a uniform surface charge density of σ. The plate is centered on, and is spinning about the z −axis at a constant angular speed ω in the counter-clock-wise sense as shown (i.e. $\vec{\omega} = \omega\hat{z}$). Find the magnetic (dipole) moment $\vec{m}$ of this spinning plate following the steps below.

NOTE: since we only have the current distribution, we can mark the location of a current element as $\vec{r}$ instead of as $\vec{r}'$ -- i.e. drop all "prime" for this problem.

(a) Find the velocity $\vec{v}$ of the current element shown in the plot (in light blue), in Cartesian coordinates and Cartesian components, and the given constants. Remember velocity is a vector.

(b) Write down the current element $dq\vec{v} = \vec{K}(\vec{r})da = \vec{K}(\vec{r})dxdy$ in Cartesian coordinates, their differentials, and Cartesian components. Remember this is a vector quantity.

(c) Calculate the integrand for $\vec{m}$ that involves a further cross-product. Use Cartesian coordinates and components. Remember cross-products are vectors.

(d) Integrate over the square plate at the moment shown (with its sides parallel to the x − and y −axes) to find the magnetic moment $\vec{m}$. Hint: remember $\vec{m}$ is a vector.

Extra Note: As it turns out, because the body has 4-fold rotational symmetry (for this property at least 3-fold is required) and $\vec{m}$ is actually a pseudo-vector – and therefore really a completely anti-symmetric 2nd order 3-tensor, $\vec{m}$ does not depend on the rotational orientation of the square around the z −axis.