1. Consider an electron in an atom with spherically symmetric potential. The energy eigenstates are $|n, l, m\rangle$ and the energy eigenvalues are $E_n = -E_0/n^2$. Now let us prepare a state at $t = 0$ as

$$|\psi\rangle = \frac{3}{5}|1, 1, 0\rangle + \frac{4}{5}|2, 1, -1\rangle$$

a. What is the energy expectation value of this state as a function of E_0? (5 points)

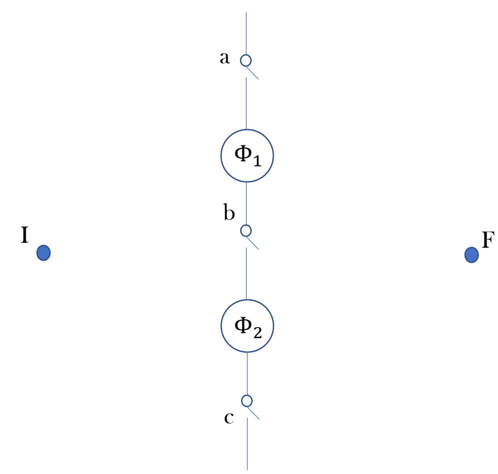
b. What is the expectation value of L_z of this state at $t = 0$? (5 points)

c. What is the expectation value of L_z of the state at $t = \frac{\hbar}{2E_0}$? (5 points)

d. If one uses L_+ operator to hit on this state at $t = 0$, what state does one get after the proper normalization? (5 points)

e. Write down the matrix representation of L_x in the eigenbasis of L_z with $l = 1$. (10 points)

2. We set an experiment to test the probabilistic interpretation in quantum mechanics, as shown in the figure. Particles, with charge $+|e|$, are emitted at point I with the same energy, and a measurement is carried at point F. The particle can reach the point F through three gates (a, b, c). There are two regions with magnetic fields pointing directly out of the page, one is between gates a and b, the other is between gates b and c. The magnetic fluxes in these two regions are given in units of $\hbar c/|e|$, where $|e|$ is the magnitude of the electron charge. More explicitly $\Phi_1 = \frac{2\pi\hbar c}{|e|} f_1$ and $\Phi_2 = \frac{2\pi\hbar c}{|e|} f_2$, with f_1 and f_2 as two free parameters.



Let us first set $f_1 = f_2 = 0$. If any one of the gates is open while the other two are closed, the probability of finding the particle at F is the same, labeled as p. If any two of the gates are open while the other one is closed, the probability of finding the particle at F is always $4p$.

a. What is the probability of finding the particle if all of the three gates are open? (7 points)

b. Now let us open gates a and b and close the gate c. Meanwhile, we set $f_1 = \frac{1}{2}$ and $f_2 = 0$. What is the probability of finding the particle at F? (7 points)

c. Now let us open gates a and c and close the gate b. Meanwhile, we set $f_1 = f_2 = \frac{1}{2}$. What is the probability of finding the particle at F? (7 points)

d. When all gates are open, what does the relation between f_1 and f_2 need to satisfy in order to get the same probability as that in problem a? (7 points)

e. Now open all gates and take $f_1 = f_2 = 1$. If we double the charge of the emitted particle while keep all other quantities unchanged, what is the probability of finding the particle at point F? (7 points)

3. Consider two different particles. Both of them have spin 1/2. They live in a system with the Hamiltonian as

$$H = c\vec{S}_1 \cdot \vec{S}_2$$

where $\vec{S}_1$ and $\vec{S}_2$ are spin operators. At $t = 0$, the particle 1 has its spin pointing down along the z-axis, and the particle 2 has its spin pointing up along the z-axis.

a. Use the commutator for components of angular momentum to explicitly prove that $(S_{1z} + S_{2z})$ is commutable with the Hamiltonian. (7 points)

b. Write the initial state at $t = 0$ in the basis of $|s, m, s_1, s_2\rangle$. Here s is the magnitudes of the total angular momentum, and m is the z component of the total angular momentum. (7 points)

c. What is the wavefunction of this state at a later time t? (7 points)

d. At a given time t, what values of energy can be measured and what are the probabilities to measurement each possible value of energy? (7 points)

e. At time t, what is the probability that the system can be found in the same state as it was at t=0? (7 points)