How big can a raindrop get?

At the meeting, you found that bigger drops fall faster. So why don't we ever see raindrops the size of golf balls?

Bring whatever you get to the next meeting: a full answer, part of an answer, or a good question. Everyone should try Part A. Parts B and C are harder.

Something new: surface tension

The surface of water acts a little like a stretched rubber skin. That's why water forms drops, and why some bugs can stand on a pond. How strong this skin is gets measured by the **surface tension**, written γ (the Greek letter "gamma"). Imagine a line 1 meter long drawn on the surface of water. The water on one side pulls on the water on the other side with a force of $\gamma \times 1$ m. For water, $\gamma \approx 0.07$ N/m.

Part A: The dripping faucet

A drop hangs from the end of a faucet. The water's "skin" grips the edge of the faucet opening all the way around. It pulls up with a force of about $\gamma \times$ (distance around the edge).

(a) The opening has a radius of about 2 mm. What is the heaviest drop that can hang there before it falls?

(b) If that drop were shaped like a ball, what would its radius be? Compare it with the 1 mm raindrop from the meeting.

(c) *Try it at home:* let a faucet drip slowly and count how many drops it takes to fill a teaspoon (about 5 mL). What does that tell you about the size of one drop?

Part B: The biggest raindrop

As a big drop falls, the air rushing past pushes hard on its bottom. That push tries to flatten the drop and tear it apart. The water's skin tries to hold it together.

(d) Surface tension squeezes a drop of radius r with an extra pressure of about γ/r. Check that γ/r has units of pressure (N/m^2).

(e) The air pushes on a falling drop with a pressure of about (density of air)$\times v^2$. Use your speed formula from Step 3 of the meeting to rewrite this pressure using r instead of v.

(f) The drop breaks apart when the air's push is stronger than the squeeze from surface tension. Estimate the radius of the biggest raindrop that can survive.

Part C: The coincidence

(g) Your answers to A and B should both be a few millimeters. But Part A has no moving air at all, and Part B is all about moving air. Why do they agree? *Hint:* in both parts, which two things are being compared?

(h) How would the answer change if you did this on the Moon? With soapy water (lower surface tension)? With liquid mercury?

Extra challenge

Falling raindrops are *not* shaped like teardrops. Based on Part B, what shape do you think a big falling raindrop has? Draw it, then look it up.

Numbers: surface tension of water ≈ 0.07 N/m density of water $= 1000$ kg/m^3 density of air $= 1.2$ kg/m^3 $g = 9.8$ m/s^2 1 N $= 1$ kg·m/s^2 mercury: surface tension ≈ 0.48 N/m, density $\approx 13{,}500$ kg/m^3