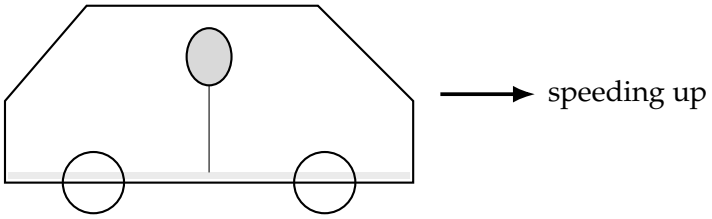
# Which way does the balloon lean?

A helium balloon is tied by a short string to the floor of a car. The windows are closed. The car is stopped, so the balloon floats straight up. Then the driver steps on the gas. Does the balloon lean forward, lean backward, or stay straight up?

---




**Before you talk to anyone:** write your prediction on the board under your name, with one sentence of reasoning.

**The plan:** Steps 1–4 are all about *water*, where it's easier to see what's going on. In Step 5 you'll bring everything you learned back to the air in the car, and answer the question.

## How tonight works

Your group will work through five problems, one step at a time. When your group finishes a step, **ask for a check-in**. The person who comes over will choose **one person** from your group to explain your answer. If they can explain it, your group gets the next step. So before you ask, make sure *everyone* in your group understands.

Each person has a job. Pass the jobs to the left at each new step. For Step 1, the Writer should be the person who has taken the *least* physics.

| | |
|---|---|
| **Writer** | Holds the *only* marker. Writes only what the group agrees on. |
| **Questioner** | Questions every step. "How do we know? Is there a case where that's wrong?" |
| **Checker** | Checks units and limiting cases. Makes sure the quietest person gets heard. |

## Three habits of physicists

1. **Draw the forces.** On *what*? Choosing the right object to draw is half the problem.
2. **Try an extreme case.** What happens if this number were zero? Or huge?
3. **Compare.** A number only means something next to something you already know.

STEP 1 OF 5

# Where does the upward push come from?

**The idea**

**Pressure** is force per area: $P = F/A$, measured in pascals ($1\ \text{Pa} = 1\ \text{N/m}^2$). A fluid at rest pushes on every surface it touches, always straight into that surface. The air around you pushes with about 100,000 Pa: that's 1 **atmosphere**.

### Your task

(a) Picture a thin, flat slab of water inside a lake: area $A$, thickness $\Delta z$, sitting still. Draw every force on it. Use the fact that it isn't moving to show that pressure grows with depth $d$ as

$$P = P_{\text{surface}} + \rho g d.$$

(b) How deep must you dive before the water adds one extra atmosphere of pressure?

(c) Now hold a cube of side $s$ under water, top face at depth $d$. Find the net force the water puts on it. Does it depend on $d$?

(d) A cube was easy. Now think about *any* shape (a rock, a fish, you). Here is a trick that avoids hard math: imagine taking the object out and filling its exact shape with water. That "water blob" just sits there. What does that tell you about the total push the surrounding water puts on it? Why does the same push act on the rock?

**Calculus toolbox: from a slab to a derivative**

Your slab gives a ratio: (change in pressure) / (thickness) $= \Delta P/\Delta z$. If you make the slab thinner and thinner, that ratio becomes the **derivative** $dP/dz$, the rate at which pressure changes with height. You'll need the derivative form in Step 5. For water, $\rho$ is the same everywhere, so the rate is constant. Pressure goes up by the same amount for every meter you go down, so you don't need calculus yet.

**Numbers you may need**

Density of water $\rho = 1000\ \text{kg/m}^3$ $\quad g = 9.8\ \text{m/s}^2$ $\quad 1\ \text{atm} \approx 1.0 \times 10^5\ \text{Pa}$

**Does your answer make sense?**

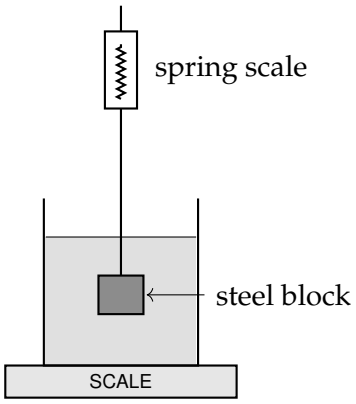
Your result in (d) is **Archimedes' principle**. Does the push depend on what the object is made of? On how deep it is? Through what point does the push act: the object's center of mass, or somewhere else?

**Extra challenge (only if your group finishes early)**

A metal cube sits on the flat bottom of a tank of water. It's machined so perfectly that *no* water gets underneath it. What force does the water put on the cube now? Why would this cube be very hard to lift?

STEP 2 OF 5

# What does the scale read?




### Your task

A beaker of water sits on a kitchen scale, which reads **1200 g**. A steel block hangs from a spring scale, which reads **790 g**. The block has a volume of 100 $cm^3$.

(a) You lower the block until it is fully under water, not touching the bottom. Before you calculate anything, each person predicts: does the kitchen scale go up, down, or stay the same?

(b) Find the new reading on *both* scales. Draw separate force diagrams for the block and for the beaker of water.

(c) Now the string is cut. The block sinks and comes to rest on the bottom. What do both scales read now?

**Numbers you may need**

Density of water $= 1$ g/$cm^3$ The scales are marked in grams: a reading of "100 g" means a force equal to the weight of 100 g.

**Does your answer make sense?**

Add the two scale readings together in each case. What should the total be, and why? Newton's third law is the key: if the water pushes up on the block, what pushes back, and on what?

**Extra challenge (only if your group finishes early)**

- While the block is sinking at a *steady* speed (after the string is cut), what does the kitchen scale read? What about in the first instant after the cut, when the block is speeding up?
- Two identical beakers with the same water level sit on the two pans of a balance. In the left one, a ping-pong ball is tied by a thread to the bottom, fully under water. In the right one, a steel ball of the same size hangs from a stand outside, fully under water. Which way does the balance tip?

STEP 3 OF 5

# Water in a speeding-up truck

### Your task

An open tank of water rides in a truck that speeds up at a steady rate $a$. After a moment the water stops sloshing and moves along with the truck.

(a) The water is now speeding up too. Think about a horizontal row of water running from the back of the tank to the front. What has to be true about the pressure at the back compared with the front?

(b) Use this to show that the water's surface tilts. Which end is higher? Show that the tilt angle $\theta$ satisfies $\tan\theta = a/g$.

(c) The truck goes from 0 to 60 mph in 6 seconds. Find $a$ and the tilt angle.

**A powerful shortcut**

Inside the truck, the water acts exactly as if gravity had changed. The new "effective gravity" is

$$\vec{g}_{\text{eff}} = \vec{g} - \vec{a},$$

which points down *and backward*. Surfaces of equal pressure are always perpendicular to $\vec{g}_{\text{eff}}$. The push from the water on anything in it is the weight of the water it pushes aside, pointing *opposite* to $\vec{g}_{\text{eff}}$.

(d) A cork is tied by a thread to the floor of the tank, fully under water. When the truck is parked, the thread is straight up and down. While the truck speeds up, which way does the thread lean, and at what angle?

(e) Now the tank has a lid and is filled to the top, with no air gap and no surface to tilt. Does the cork still lean? Why?

**Numbers you may need**

1 mph $\approx$ 0.447 m/s  60 mph $\approx$ 26.8 m/s  $g = 9.8\ \text{m/s}^2$

**Does your answer make sense?**

A small fishing weight hangs on a thread from a bar above the tank, under water. Which way does *it* lean? Why do the cork and the weight go opposite ways?

**Extra challenge (only if your group finishes early)**

An ice cube floats in a glass of water inside an elevator that is speeding up as it goes up. Does the ice ride higher, lower, or the same as when the elevator is still? Does the pressure at the bottom of the glass change?

STEP 4 OF 5

# The spinning bucket

### Your task

A bucket of water spins at a steady rate $\omega$ (in radians per second) around its vertical center line. After a while the water spins along with it.

(a) A bit of water a distance $r$ from the center moves in a circle. What is its acceleration (size and direction)? So which way does $\vec{g}_{\text{eff}}$ point at that spot?

(b) The surface must be perpendicular to $\vec{g}_{\text{eff}}$ everywhere. Find the slope of the surface, $dz/dr$, at distance $r$. Then find the height of the surface, $z(r)$.

(c) A bucket 30 cm across spins once per second. How much higher is the water at the edge than at the center?

(d) A cork and a small pebble are dropped into the spinning water. Which way does each one drift, toward the center or toward the wall?

(e) Steps 3 and 4 together: write a one-sentence rule. *In a fluid that feels effective gravity $\vec{g}_{\text{eff}}$, an object less dense than the fluid moves* _____, *and an object more dense moves* _____.

**Calculus toolbox: from slope to shape**

*Same math, different problem.* A ball dropped from rest has velocity $dy/dt = -gt$. To find its height, ask: what function has derivative $-gt$? Answer: $y = y_0 - \frac{1}{2}gt^2$. Check by taking the derivative. Your slope $dz/dr$ works the same way.
*No calculus?* Find the slope at $r = 0,\ 5,\ 10,\ 15$ cm. Between neighboring points, rise $\approx$ (average slope) $\times$ (5 cm). Add up the rises from the center to the edge, then sketch the shape through your points.

**Numbers you may need**

Circular motion: acceleration $= \omega^2 r$, toward the center. Once per second: $\omega = 2\pi \approx 6.3$ rad/s $g = 9.8\ \text{m/s}^2$

**Does your answer make sense?**

Does the shape depend on the density of the liquid? What does your formula give when $\omega \to 0$?

**Extra challenge (only if your group finishes early)**

- Some telescopes use a spinning pool of liquid mercury as the mirror. A parabola $z = r^2/(4f)$ focuses light at height $f$. How long should one turn take to make a mirror with $f = 8$ m?
- Doctors spin blood in a *centrifuge* to separate heavy red blood cells from the lighter liquid. Use your rule from (e) to explain how it works.

STEP 5 OF 5

# Back to the car

Your rule from Step 4 came from water. Air is different in two ways: it is about 800 times less dense, and it *compresses*. Squeeze air and it gets denser: at a fixed temperature $T$, the **ideal gas law** gives $\rho = PM/RT$, where $M$ is the mass of one mole of gas. Do either of these differences break your rule?

### Your task

(a) In Step 1 you found $dP/dz = -\rho g$ ($z$ measured upward). For air, $\rho$ depends on $P$. Solve for $P(z)$, assuming the temperature is the same at every height. Your answer contains a length $H$. Find $H$ for Earth's air. Salt Lake City is 1.3 km up: what fraction of sea-level pressure do we have?

(b) A car is about 1.5 m tall. By what fraction does the air's density change from floor to roof? Over the size of a car, does air act like a compressible fluid or like a "thin water"?

(c) Look back at Steps 3 and 4. Did the tilt angle or your rule depend on how dense the fluid is?

(d) **Now answer the opening question.** Which way does the balloon lean, and at what angle, for a car that goes 0 to 60 mph in 6 s?

(e) Explain it without the words "effective gravity." What does the air in the car do when the car speeds up? Estimate how much higher the pressure is at the back of the car than at the front (about 2 m apart).

**Calculus toolbox: when the rate depends on the amount**

*Same math, different problem.* A radioactive sample loses atoms at a rate proportional to how many it has: $dN/dt = -N/\tau$. Separate the variables: $dN/N = -dt/\tau$. Integrate: $\ln N = -t/\tau + C$, so $N = N_0\, e^{-t/\tau}$. After substituting $\rho = PM/RT$, your equation for $P$ has exactly this form.
*No calculus?* Climb 1 km at a time, with $\Delta P \approx -\rho g\,(1000\text{ m})$ using the density at the bottom of each step. Show each step multiplies $P$ by about 0.88. Find $P$ at 1, 2, 3, 5 km. What kind of function does that?

**Numbers you may need**

$R = 8.31$ J/(mol·K) Air: $M = 0.029$ kg/mol $T \approx 288$ K Sea level: $P \approx 1.0 \times 10^5$ Pa, $\rho \approx 1.2$ kg/m$^3$

**Does your answer make sense?**

Fuzzy dice hang from the mirror of the same car. Which way do they swing? Compare with your cork and fishing weight from Step 3. How did your prediction on the board do?

**Extra challenge (only if your group finishes early)**

Outside, you let go of a stretchy rubber balloon and a foil balloon that can't stretch. Where does each stop rising? If a gust lifts the foil one a little, does it come back?

WHOLE-ROOM DISCUSSION

# A balloon for Venus

The surface of Venus is about 460 °C, and its air pressure is about 90 times ours. No lander has lasted more than about two hours there. But about 55 km up, inside the clouds, the temperature is close to room temperature and the pressure is about half of Earth's at sea level.

Engineers at NASA's Jet Propulsion Laboratory have built and flown a small prototype **aerobot**: a robot balloon that could float in Venus's clouds and change its height on command. It is a balloon inside a balloon:

- The **outer** balloon is thin and floppy. Its helium stays at the same pressure as the air outside.
- The **inner** balloon is strong and barely stretches, like a foil party balloon. A pump can push helium from the outer balloon into the inner one, squeezing it to high pressure. A valve can let it back out.

In 2022 the prototype flew twice over the Nevada desert, at about 2.5 km above sea level.

1. Pumping helium *into* the inner balloon makes the aerobot sink. No helium leaves. Why does it sink?
2. Why test the prototype at 2.5 km above Nevada, and not at sea level? (Hint: the air's density at that height is about the same as Venus's at 55 km. Use your Step 5 formula to check.)
3. Venus's air is mostly carbon dioxide, with $M = 0.044$ kg/mol. Earth's air has $M = 0.029$ kg/mol. Someone suggests a balloon filled with ordinary *breathable air* would float on Venus. Is that true? How does its lift compare with helium's ($M = 0.004$ kg/mol)?
4. The Sun heats the balloon during the Venus day. What happens to its height? How could the aerobot fight back?