Which way does the balloon lean?

Forward. A helium balloon in a car that speeds up leans *forward*, about 24° for a car that goes from 0 to 60 mph in 6 seconds. These pages show how each step builds toward that answer.

The big picture. Buoyancy is not a separate kind of force. It is what you get when a fluid's pressure is higher on one side of an object than the other. Pressure in a fluid always lines up with whatever "gravity" the fluid feels. When a car speeds up, the air inside it feels a gravity that points down *and backward*. A balloon is lighter than the air around it, so it moves the opposite way: up *and forward*.

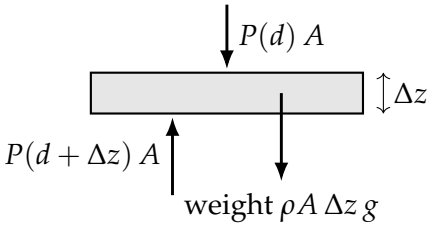
Use these solutions to check your work, not to replace it. If your group got stuck partway, try the next part yourself before reading the answer.

Step 1: Where does the upward push come from?

(a) Picture a flat slab of water with area A and thickness Δz, with its top at depth d. Three forces act on it:

- The water above pushes **down** on the top face: $P(d)\,A$.
- The water below pushes **up** on the bottom face: $P(d+\Delta z)\,A$.
- Gravity pulls **down** on the slab's mass, which is $\rho A\,\Delta z$: weight $= \rho A\,\Delta z\,g$.

The water around the slab's edges pushes sideways from every direction, and those pushes cancel.



The slab isn't moving, so the forces balance:

$$P(d+\Delta z)\,A = P(d)\,A + \rho A\,\Delta z\,g \quad\Longrightarrow\quad P(d+\Delta z) - P(d) = \rho g\,\Delta z.$$

Every extra bit of depth Δz adds the same pressure, $\rho g\,\Delta z$. Stack slabs from the surface down to depth d and the pressure adds up to

Answer: $P = P_{\text{surface}} + \rho g d$.

Toolbox note: dividing by Δz gives $\Delta P/\Delta z = \rho g$. As the slab gets thinner, this ratio is the derivative. Measured with z pointing *up*, it reads $dP/dz = -\rho g$. You need this form in Step 5.

(b) We need $\rho g d = 1.0 \times 10^5$ Pa:

$$d = \frac{1.0 \times 10^5}{1000 \times 9.8} \approx 10 \text{ m}.$$

Answer: about 10 m. Every 10 m of water adds one atmosphere. That's why your ears hurt at the bottom of a deep pool.

(c) The four side faces come in opposite pairs at the same depths, so their pushes cancel. That leaves the top and bottom:

$$\begin{aligned} \text{down on top:} &\quad (P_{\text{surface}} + \rho g d)\, s^2 \\ \text{up on bottom:} &\quad (P_{\text{surface}} + \rho g (d+s))\, s^2 \end{aligned}$$

Subtract. The P_{surface} terms and the $\rho g d$ terms cancel:

Answer: net upward force $= \rho g s^3$. It does **not** depend on the depth d. Since s^3 is the cube's volume, $\rho g s^3$ is the weight of a cube of water.

(d) The "water blob" is water shaped exactly like the rock, sitting still in the lake. Two forces act on it: its own weight, and the push from all the water around it. It isn't moving, so the push must exactly cancel the weight:

$$\text{push from surrounding water} = \rho_{\text{water}}\, V g, \quad \text{straight up.}$$

Now swap the blob for the rock. The water around it is in exactly the same place, at exactly the same pressures. It can't tell the difference, so it pushes on the rock the same way.

Answer: the push on *any* object equals the weight of the water it pushes aside: $F = \rho_{\text{water}} V g$. This is **Archimedes' principle**.

Does your answer make sense? The push doesn't depend on what the object is made of; only its volume matters. It doesn't depend on depth either, as long as the water doesn't compress. And it acts through the center of the *water blob* (the "center of buoyancy"), not the object's own center of mass. If it acted anywhere else, the blob would start to spin on its own, and still water doesn't do that.

Extra challenge (sealed cube). With no water underneath, there is nothing to push up. The only push from the water is *down* on the top:

$$F = (P_{\text{surface}} + \rho g d)\, s^2.$$

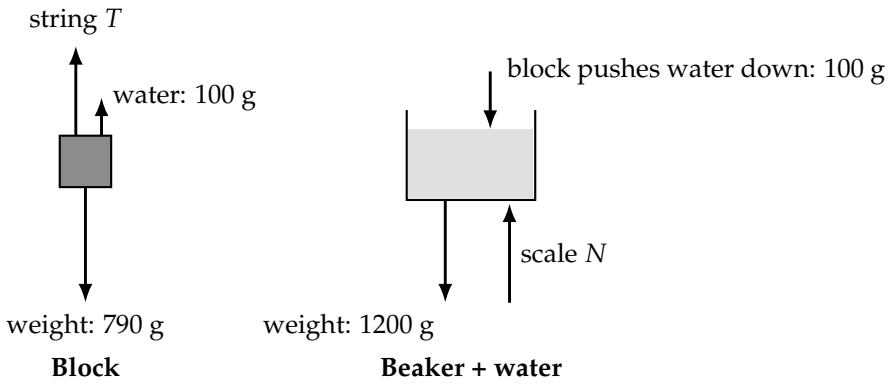
For a 10 cm cube 1 m deep: $(1.0 \times 10^5 + 9800) \times 0.01 \approx 1100$ N, about the weight of 110 kg. You'd have to lift that plus the cube. A suction cup works the same way. As soon as a little water sneaks underneath, the push from below comes back and the cube lifts easily.

Step 2: What does the scale read?

Key numbers. The block's volume is 100 cm^3, so it pushes aside 100 cm^3 of water, which has a mass of 100 g. The water pushes up on the block with a force equal to the weight of 100 g.

(a) The kitchen scale goes **up**. Most people guess "the same," because the block isn't touching the beaker.

(b) Draw two separate force diagrams.



Block: it isn't moving, so $T + 100 = 790$, and $T = 690$ g.
Beaker + water: by Newton's third law, if the water pushes *up* on the block with 100 g, the block pushes *down* on the water with 100 g. So $N = 1200 + 100 = 1300$ g.

Answer: spring scale **690 g**, kitchen scale **1300 g**.

(c) Now the beaker, the water, and the block all rest on the kitchen scale, and nothing else holds any of them up.

Answer: spring scale **0**, kitchen scale $1200 + 790 =$ **1990 g**.

Does your answer make sense? Add the two readings: $690 + 1300 = 1990$ in (b), and $0 + 1990 = 1990$ in (c). The two scales together hold up everything: beaker, water, and block. Lowering the block into the water only changes *which* scale carries the 100 g. The water's push on the block moves 100 g of weight from the spring scale to the kitchen scale.

Extra challenge 1 (sinking block). At *steady* speed, the block isn't speeding up, so the water pushes up on it with the full 790 g: 100 g of buoyancy plus 690 g of drag from moving through the water. By the third law, the block pushes down on the water with 790 g, and the scale reads **1990 g**, the same as when the block rests on the bottom. Just after the cut, the block is barely moving, so there's no drag yet. The water only pushes up with 100 g, and the scale reads about **1300 g**. As the block speeds up, drag grows and the reading climbs toward 1990 g. Whenever something in the system is speeding up downward, the scale reads less than the total weight.

Extra challenge 2 (ping-pong vs. steel). *Left:* the ping-pong ball and its thread are inside the beaker, so the pan holds beaker, water, and ball. The ball weighs almost nothing. *Right:* the steel ball hangs from a stand, so its weight doesn't count. But the water pushes up on it, so it pushes down on the water with the weight of the water it displaces.

Answer: the balance tips toward the **steel** side, by about the weight of one ball's volume of water.

Step 3: Water in a speeding-up truck

(a) Think of a horizontal row of water from the back of the tank to the front, with length L and cross-section area A. Its mass is ρAL. It speeds up along with the truck, so something must push it forward with a force $\rho AL\,a$. The only horizontal forces on it are the pressures at its two ends:

$$(P_{\text{back}} - P_{\text{front}})\,A = \rho AL\,a \quad \Longrightarrow \quad P_{\text{back}} - P_{\text{front}} = \rho a L.$$

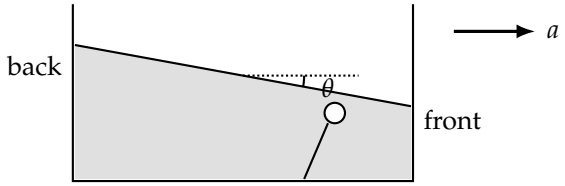
Answer: the pressure is **higher at the back**, by $\rho a L$.

(b) Pick any height in the water. From Step 1, the pressure there is $P_{\text{air}} + \rho g \times$(depth of water above it). The pressure is higher at the back, so there must be *more water above* that height at the back. The extra depth is

$$\frac{\rho a L}{\rho g} = \frac{aL}{g}.$$

So the surface rises by aL/g over a distance L:

$$\tan\theta = \frac{\text{rise}}{\text{run}} = \frac{aL/g}{L} = \frac{a}{g}.$$



Surface higher at the back. Cork leans forward.

Answer: the surface is **higher at the back**, tilted at $\tan\theta = a/g$.

(c) $a = \dfrac{26.8\ \text{m/s}}{6\ \text{s}} \approx 4.5\ \text{m/s}^2$. Then $\tan\theta = 4.5/9.8 \approx 0.46$.

Answer: $a \approx 4.5\ \text{m/s}^2$ and $\theta \approx 24^\circ$.

(d) The cork has volume V and mass m. It is less dense than water, so $m < \rho V$. The water pushes on the cork exactly as it would push on a blob of water the same shape (Step 1's trick). That blob would be moving along with the rest of the water, so the push on it must do two jobs:

- hold it up: $\rho V g$, upward;
- speed it up with the truck: $\rho V a$, forward.

The cork gets this same push, but it's lighter than the blob. It only needs mg up and ma forward. The extra, $(\rho V - m)g$ up and $(\rho V - m)a$ forward, has to be held back by the thread. So the thread pulls the cork *down and backward*, which means the cork sits *up and forward* of where the thread is tied. The angle from vertical is

$$\tan\theta = \frac{(\rho V - m)\,a}{(\rho V - m)\,g} = \frac{a}{g}.$$

Answer: the thread leans **forward** at 24°, the same angle as the water's surface.

(e) Yes. The higher pressure at the back is in the water everywhere, whether or not there's a surface. The tilted surface only makes the pressure pattern visible. A sealed tank filled to the top is just like a sealed car filled with air. You'll come back to this in Step 5.

Does your answer make sense? The fishing weight is *denser* than water, so $m > \rho V$. The water's push isn't enough to hold it up or to keep up with the truck. The thread has to pull it up and forward, so the weight hangs **back** at 24°. Both threads line up along the same tilted direction. The light object points "up" along it and the heavy one points "down."

Extra challenge (elevator). In an elevator speeding up while going up, everything acts as if gravity were stronger: $g + a$ instead of g. The ice's weight and the water's push both get stronger by the same factor, so the ice floats at **the same level**. The pressure at the bottom of the glass **goes up**: it becomes $\rho(g + a)h$.

Step 4: The spinning bucket

(a) The water moves in a circle, so it speeds up toward the center: $a = \omega^2 r$, pointing **inward**. Then $\vec{g}_{\text{eff}} = \vec{g} - \vec{a}$ has g pointing down and $\omega^2 r$ pointing **outward**.

Answer: $\vec{g}_{\text{eff}}$ points down and outward, toward the wall.

(b) The surface must be perpendicular to $\vec{g}_{\text{eff}}$. Draw $\vec{g}_{\text{eff}}$ at one spot. Its outward part is $\omega^2 r$ and its downward part is g. A line perpendicular to it rises by $\omega^2 r$ for every g it moves outward, so

$$\frac{dz}{dr} = \frac{\omega^2 r}{g}.$$

To get the height, ask what function has derivative $(\omega^2/g)\, r$. Just as velocity $-gt$ gives height $-\frac{1}{2}gt^2$ for a falling ball:

Answer: $z(r) = \dfrac{\omega^2 r^2}{2g}$, measured from the center of the surface. This is a parabola spun around the axis (a "paraboloid").

(c) $r = 0.15$ m and $\omega = 2\pi \approx 6.3$ rad/s:

$$z = \frac{(6.3)^2 (0.15)^2}{2 \times 9.8} \approx \frac{39.5 \times 0.0225}{19.6} \approx 0.045 \text{ m}.$$

Toolbox (no-calculus) route: the slope is $\omega^2 r/g \approx 4.0\, r$ (with r in meters).

r (cm)	slope	average slope over the next 5 cm	rise over the next 5 cm
0	0	0.10	0.5 cm
5	0.20	0.30	1.5 cm
10	0.40	0.50	2.5 cm
15	0.60		
		total rise	**4.5 cm**

Answer: about **4.5 cm** higher at the edge.

(d) The pressure gets higher toward the wall, just as it got higher toward the back of the truck. The water pushes on any object the same way it pushes on a water blob the same shape. That blob needs an inward push of $\rho V \omega^2 r$ to go in its circle. The cork is lighter than the blob, so this push is more than it needs: the extra pushes it **inward**. The pebble is heavier, so the push isn't enough to keep it in its circle and it drifts **outward** (while sinking).

Answer: cork toward the **center**, pebble toward the **wall**.

(e) Steps 3 and 4 point to one rule.

Answer: An object less dense than the fluid moves **opposite** to $\vec{g}_{\text{eff}}$ (toward "up"). An object more dense moves **along** $\vec{g}_{\text{eff}}$ (toward "down").

In the truck, "up" tilts forward. In the bucket, "up" tilts toward the center.

Does your answer make sense? The density ρ never shows up in $z = \omega^2 r^2/(2g)$, so honey or mercury would make the same shape. When $\omega \to 0$, $z \to 0$: a flat surface, as it should be for a bucket at rest.

Extra challenge (mercury mirror). Match the shapes: $\dfrac{\omega^2 r^2}{2g} = \dfrac{r^2}{4f}$ gives $f = \dfrac{g}{2\omega^2}$. For $f = 8$ m:

$$\omega = \sqrt{\frac{9.8}{16}} \approx 0.78 \text{ rad/s}, \qquad \text{time for one turn} = \frac{2\pi}{0.78} \approx 8 \text{ s}.$$

That's slow. A gentle, very steady spin makes a near-perfect mirror.

Extra challenge (centrifuge). Spinning makes $\vec{g}_{\text{eff}}$ point outward and become much stronger than ordinary gravity. Red blood cells are denser than the liquid around them, so by the rule in (e) they move outward, to the bottom of the tube. The lighter liquid stays toward the center. Gravity alone would do the same thing, but far more slowly.

Step 5: Back to the car

(a) Put $\rho = PM/RT$ into $dP/dz = -\rho g$:

$$\frac{dP}{dz} = -\frac{Mg}{RT}\,P = -\frac{P}{H}, \qquad \text{where } H = \frac{RT}{Mg}.$$

This has the same form as the toolbox's $dN/dt = -N/\tau$. Separate the variables and integrate:

$$\frac{dP}{P} = -\frac{dz}{H} \implies \ln P = -\frac{z}{H} + C \implies P = P_0\, e^{-z/H}.$$

$$H = \frac{8.31 \times 288}{0.029 \times 9.8} \approx 8400 \text{ m}.$$

For Salt Lake City, $P/P_0 = e^{-1.3/8.4} \approx 0.86$.

Answer: $P = P_0 e^{-z/H}$ with $H \approx 8.4$ km (the **scale height**). Salt Lake City has about **86%** of sea-level pressure.

Toolbox (no-calculus) route: climb 1 km. The pressure drops by ρg (1000 m), and since $\rho = PM/RT$,

$$\Delta P = -\frac{PMg}{RT}(1000) = -P \times \frac{0.029 \times 9.8 \times 1000}{8.31 \times 288} \approx -0.12\,P.$$

Every kilometer the pressure loses about 12% of what it had, so it gets multiplied by about 0.88:

height (km)	0	1	2	3	5	8.4
pressure (atm), steps of $\times 0.88$	1	0.88	0.77	0.68	0.53	0.34
pressure (atm), exact $e^{-z/H}$	1	0.89	0.79	0.70	0.55	0.37

Multiplying by the same number at every step is exactly what an exponential does. The steps run a little low because each one uses the density at the *bottom* of the step, which is slightly too large. Smaller steps fix this.

(b) Density is proportional to pressure, so over a height h the fractional change is about h/H:

$$\frac{1.5\text{ m}}{8400\text{ m}} \approx 0.0002 = 0.02\%.$$

Answer: about **0.02%**. Over the size of a car, air hardly compresses at all. It acts like a very thin water.

(c) Look back at the formulas.

Answer: No. The tilt was $\tan\theta = a/g$, and the rule from Step 4 only compares the object's density with the fluid's. Neither depends on how dense the fluid is.

So the rule works for air just as well as for water.

(d) Inside the car, $\vec{g}_{\text{eff}}$ points down and backward, tilted 24° from vertical. A helium balloon is less dense than air, so it goes toward "up," opposite to $\vec{g}_{\text{eff}}$.

Answer: the balloon leans **forward** at about 24°, exactly like the cork in Step 3.

(e) When the car speeds up, the air inside gets left behind a little and piles up toward the back, like the water in the truck. That makes the pressure a bit higher at the back. The difference from back to front (2 m apart) is

$$\rho a L \approx 1.2 \times 4.5 \times 2 \approx 11\text{ Pa}.$$

That's only about 1/10,000 of an atmosphere, but it's enough. For comparison, the pressure difference from floor to roof is $\rho g h \approx 1.2 \times 9.8 \times 1.2 \approx 14$ Pa, and that is what makes the balloon float at all. The higher pressure at the back pushes the balloon forward. The balloon is lighter than the air it pushes aside, so this push wins over its tendency to lag behind.

Answer: the air sloshes back; the pressure at the back is higher by about 11 Pa, and that pushes the balloon forward.

Does your answer make sense? The fuzzy dice are denser than air, so they swing **backward** at 24°, like the fishing weight in Step 3. Balloon and dice stay parallel to each other, pointing opposite ways. How did your prediction on the board do? Most people guess backward for the balloon too.

Extra challenge (two balloons outside). *Foil balloon:* its volume V stays fixed. It rises until the air is thin enough that the push, $\rho_{\text{air}} V g$, equals its weight, then stops. If a gust lifts it, the air there is thinner, the push is smaller than its weight, and it sinks back. If it drops, the push grows and it rises back. This is **stable**. *Rubber balloon:* its helium stays at about the same pressure as the air outside, so as it rises and the outside pressure falls, the balloon swells. The thinner air times the bigger volume stays about the same, so the push hardly changes. Nothing stops it from rising until the rubber stretches too far and the balloon pops.

Discussion: A balloon for Venus

1. Why does pumping helium into the inner balloon make it sink? The pump squeezes helium into the strong inner balloon at high pressure, where it takes up less room. The outer balloon goes slack, so the whole aerobot takes up less volume. It pushes aside less of Venus's air, so the push on it gets smaller. Its mass hasn't changed, so it sinks. It sinks until it reaches denser air where the push balances its weight again. Letting helium back out through the valve does the reverse.

2. Why test at 2.5 km above Nevada? How much a balloon can lift and how it changes shape both depend on the density of the air around it. Near 55 km, Venus's air has a density of about 1 kg/m^3. On Earth,

using the Step 5 formula, air thins from 1.2 kg/m^3 at sea level to about 0.95 kg/m^3 near

$$z = H \ln \frac{1.2}{0.95} \approx 8.4 \text{ km} \times 0.23 \approx 2 \text{ km}.$$

The real flights went to 2.4–2.8 km. Our estimate assumes the temperature is the same at every height, which isn't quite true, so it comes out a little low.

3. Would breathable air float on Venus? Yes. At the same pressure and temperature, a mole of any gas takes up the same volume. One mole of Venus air (CO_2) has a mass of 44 g. A balloon holding one mole of breathable air (29 g) pushes aside 44 g of Venus air, giving $44 - 29 = 15$ g of lift. One mole of helium (4 g) gives $44 - 4 = 40$ g.

Answer: breathable air gives $15/40 = 3/8$ of helium's lift. That's less, but you could live inside the balloon. Some people have proposed floating research stations on Venus for exactly this reason.

4. What does sunlight do? Heating makes the gas in the floppy outer balloon expand, so the aerobot takes up more volume, gets a bigger push, and rises. To fight back, it pumps helium into the inner balloon, which shrinks the total volume and brings it back down. At night the gas cools and the valve lets helium back out. The pump and valve are like a fish adjusting its swim bladder.

Sources for the Venus page: J. S. Izraelevitz et al., "Flight Demonstration and Model Validation of a Prototype Variable-Altitude Venus Aerobot," arXiv:2411.06643; NASA JPL Robotics, "Venus Variable-Altitude Aerobots."