Why don’t clouds fall?

These are worked answers to the five steps from the meeting, including the extra challenges. Your numbers don’t need to match ours exactly. We rounded the same way the cards asked you to ($\pi \approx 3$, $g \approx 10\ \mathrm{m/s^2}$ when it helps), so if your answer is within a factor of about 2 of ours, you did it right.

If you got a different answer, don’t just copy ours. Find the line where your work and ours split apart. That line is where the learning is.

The big picture in one paragraph

A simple formula built from units gave a good speed for a raindrop (Step 3). The same formula failed badly for a cloud drop (Step 4). That failure pointed to something we had left out, the stickiness of air, and the new formula worked for cloud drops but failed for raindrops. A single number, the Reynolds number, tells you which formula to trust (Step 5). Clouds do fall, but so slowly that the rising air inside them easily holds them up.

STEP 1 OF 5 · SOLUTIONS

How heavy is a raindrop?

(a) Volume

Change to meters first: $r = 1\text{ mm} = 10^{-3}\text{ m}$, so $r^3 = 10^{-9}\text{ m}^3$.

$$V = \tfrac{4}{3}\pi r^3 \approx 4 \times 10^{-9}\text{ m}^3.$$

(With a calculator, $4.2 \times 10^{-9}\text{ m}^3$.)

(b) Mass

$$m = \rho_{\text{water}}\, V \approx (1000\text{ kg/m}^3)(4 \times 10^{-9}\text{ m}^3) = 4 \times 10^{-6}\text{ kg}.$$

Since $1\text{ mg} = 10^{-6}\text{ kg}$, that's about **4 mg**. A grain of rice is about 20 mg, so a raindrop weighs about a fifth as much.

(c) Weight

$$W = mg \approx (4 \times 10^{-6}\text{ kg})(10\text{ m/s}^2) = 4 \times 10^{-5}\text{ N}.$$

Does your answer make sense?

A 1-liter bottle holds 10^{-3} m^3. Number of drops:

$$\frac{10^{-3}\text{ m}^3}{4 \times 10^{-9}\text{ m}^3} \approx 250{,}000.$$

A quarter of a million drops to fill a bottle. That's believable: think of how long a light rain takes to fill a bucket.

Common trap: 1 mm^3 is *not* 10^{-3} m^3. You have to cube the conversion too: $(10^{-3})^3 = 10^{-9}$.

Extra challenge

A cloud drop has $r = 10\ \mu\text{m} = 0.01$ mm. A raindrop's radius is $1\text{ mm} \div 0.01\text{ mm} = 100$ times bigger. Volume goes like r^3, so the raindrop's volume is $100^3 = 1{,}000{,}000$ times bigger. **About a million cloud drops** have to join to make one raindrop.

Falling with no air

(a) Speed at the ground

$$v = \sqrt{2gh} \approx \sqrt{2 \times 10 \times 2000}\ \text{m/s} = \sqrt{40{,}000}\ \text{m/s} = 200\ \text{m/s}.$$

In mph: $200 \times 2.2 \approx 440$ mph. That's more than twice as fast as a jet taking off.

Where the formula comes from (if you were curious). Energy is conserved. The drop starts with gravitational energy mgh and no motion. It ends with motion energy $\frac{1}{2}mv^2$. Setting $mgh = \frac{1}{2}mv^2$ and solving gives $v = \sqrt{2gh}$.

(b) Does the mass matter?

No. There's no m in $v = \sqrt{2gh}$. (In the energy argument above, the m cancels from both sides.) With no air, a raindrop, a bowling ball, and a feather all hit the ground at the same speed.

Does your answer make sense?

Rain at 440 mph would hurt a lot, and could break windows. Real rain doesn't, so something is missing. **The missing piece is the air.** Air pushes back on anything moving through it.

Extra challenge

Time with no air: $h = \frac{1}{2}gt^2$, so $t = \sqrt{2h/g} = \sqrt{4000/10}\ \text{s} = \sqrt{400}\ \text{s} = 20\ \text{s}$.

Real raindrop: 5 minutes = 300 s. Average speed = 2000 m ÷ 300 s ≈ 7 m/s. That's about 30 times slower than the no-air answer. Air matters a lot.

The air pushes back

(a) What could change the air's push?

Good answers include: how fast the drop moves, how big it is, its shape, how dense the air is, how "sticky" or thick the air is, temperature, and wind. There's no single right list. Keep yours: one item on it shows up again in Step 4.

(b) Build a force from the units

The ingredients: ρ_{air} in $\dfrac{\text{kg}}{\text{m}^3}$, v in $\dfrac{\text{m}}{\text{s}}$, r in m. Target: $\dfrac{\text{kg}\cdot\text{m}}{\text{s}^2}$.

- **kg:** a force has exactly one kg. Only ρ_{air} has kg, so it appears once: ρ_{air}^1.
- **s:** a force has $1/\text{s}^2$. Only v has seconds, so it appears squared: v^2.
- **m:** so far $\dfrac{\text{kg}}{\text{m}^3}\times\dfrac{\text{m}^2}{\text{s}^2}=\dfrac{\text{kg}}{\text{m}\cdot\text{s}^2}$. We need m^{+1}, so we need two more meters: r^2.

$$\text{push from the air} \approx \rho_{\text{air}}\, v^2\, r^2.$$

Every power was forced. There was only one way to do it.

(c) Solve for the speed

Weight $\approx \rho_{\text{water}}\, r^3 g$ (leaving out the $\frac{4}{3}\pi$). Set push = weight:

$$\rho_{\text{air}}\, v^2 r^2 = \rho_{\text{water}}\, r^3 g \quad\Longrightarrow\quad v^2 = \frac{\rho_{\text{water}}\, g\, r}{\rho_{\text{air}}} \quad\Longrightarrow\quad v \approx \sqrt{\frac{\rho_{\text{water}}\, g\, r}{\rho_{\text{air}}}}.$$

Two of the three r's canceled. Bigger drops fall faster, but only like $\sqrt{r}$.

(d) The 1 mm raindrop

$$v \approx \sqrt{\frac{(1000)(9.8)(10^{-3})}{1.2}}\ \text{m/s} = \sqrt{8.2}\ \text{m/s} \approx 3\ \text{m/s}.$$

Does your answer make sense?

The measured speed is 6.5 m/s, and the Step 2 challenge gave about 7 m/s. We're off by about a factor of 2. **That's very good.** We started with almost nothing and landed within a factor of 2. Going from 200 m/s (no air) to 3 m/s was the hard part. The units trick can't find plain numbers like $\frac{4}{3}\pi$, and that's where most of the missing factor hides: putting $\frac{4}{3}\pi \approx 4$ back into the weight multiplies v by $\sqrt{4}=2$, giving about 6 m/s.

Extra challenge

4 times the radius: v goes like $\sqrt{r}$, so the speed goes up by $\sqrt{4}=2$. It falls twice as fast.

Hailstone, $r = 2$ cm $= 0.02$ m, density 900 kg/m^3:

$$v \approx \sqrt{\frac{(900)(9.8)(0.02)}{1.2}}\ \text{m/s} = \sqrt{147}\ \text{m/s} \approx 12\ \text{m/s}.$$

Put the $\frac{4}{3}\pi$ back in and you get about 25 m/s, roughly 55 mph. Real large hail falls at about 20 to 40 m/s, which is why it dents cars.

When the formula fails

(a) The Step 3 formula on a cloud drop

$r = 10^{-5}$ m:

$$v \approx \sqrt{\frac{(1000)(9.8)(10^{-5})}{1.2}}\ \text{m/s} = \sqrt{0.082}\ \text{m/s} \approx 0.3\ \text{m/s}.$$

The measured speed is 0.01 m/s. We're **30 times too fast**. That isn't a rounding problem. The formula is missing some physics.

(b) Build a force from stickiness

The ingredients: η in $\dfrac{\text{kg}}{\text{m}\cdot\text{s}}$, v in $\dfrac{\text{m}}{\text{s}}$, r in m.

- **kg:** only η has kg, so it appears once.
- **s:** η already has one $1/\text{s}$. We need one more, so v^1. Now $\dfrac{\text{kg}}{\text{m}\cdot\text{s}} \times \dfrac{\text{m}}{\text{s}} = \dfrac{\text{kg}}{\text{s}^2}$.
- **m:** we need one meter, so r^1.

$$\text{push from the air} \approx \eta\, v\, r.$$

Notice this push grows like v, not v^2. Doubling the speed only doubles the push.

(c) Solve for the speed

$$\eta\, v\, r = \rho_{\text{water}}\, r^3 g \quad \Longrightarrow \quad v \approx \frac{\rho_{\text{water}}\, g\, r^2}{\eta}.$$

Now the speed grows like r^2, much faster than before.

(d) The cloud drop

$$v \approx \frac{(1000)(9.8)(10^{-10})}{1.8\times 10^{-5}}\ \text{m/s} = \frac{9.8\times 10^{-7}}{1.8\times 10^{-5}}\ \text{m/s} \approx 0.05\ \text{m/s} = 5\ \text{cm/s}.$$

Measured: 1 cm/s. We're off by about 5, much better than 30. (The full textbook formula, called Stokes' law, has extra plain numbers in it and gives about 1.2 cm/s.)

Does your answer make sense?

New formula on the 1 mm raindrop, $r = 10^{-3}$ m:

$$v \approx \frac{(1000)(9.8)(10^{-6})}{1.8\times 10^{-5}}\ \text{m/s} \approx 540\ \text{m/s}.$$

That's faster than the 200 m/s from Step 2, where there was *no air at all*. Air can only slow a drop down, so this answer is impossible. The stickiness formula fails for raindrops.

So which formula when? The density formula (Step 3) works for big, fast drops. The stickiness formula (Step 4) works for small, slow drops. Step 5 makes "big" and "small" precise.

Extra challenge

Time = distance ÷ speed = 1000 m ÷ 0.01 m/s = 100,000 s. Divide by 3600 s per hour: **about 28 hours**, more than a full day to fall 1 km in perfectly still air.

STEP 5 OF 5 · SOLUTIONS

Which formula wins?

(a) Divide the two pushes

$$\frac{\rho_{\text{air}}\, v^2 r^2}{\eta\, v\, r} = \frac{\rho_{\text{air}}\, v\, r}{\eta}.$$

Units: $\dfrac{\dfrac{\text{kg}}{\text{m}^3}\cdot\dfrac{\text{m}}{\text{s}}\cdot\text{m}}{\dfrac{\text{kg}}{\text{m}\cdot\text{s}}} = \dfrac{\text{kg}/(\text{m}\cdot\text{s})}{\text{kg}/(\text{m}\cdot\text{s})}$. **Everything cancels.** The ratio is a plain number with no units.

(b) Values

Raindrop: $\dfrac{(1.2)(6.5)(10^{-3})}{1.8\times10^{-5}} \approx 430.$ Cloud drop: $\dfrac{(1.2)(0.01)(10^{-5})}{1.8\times10^{-5}} \approx 0.007.$

(c) Which formula to trust

Ratio much bigger than 1: the density push is the big one, so use the Step 3 formula. That's the raindrop.
Ratio much smaller than 1: the stickiness push is the big one, so use the Step 4 formula. That's the cloud drop.

(d) Where the formulas agree

Set the two speeds equal:

$$\sqrt{\frac{\rho_{\text{water}}\, g\, r}{\rho_{\text{air}}}} = \frac{\rho_{\text{water}}\, g\, r^2}{\eta}.$$

Square both sides, then cancel one r and one $\rho_{\text{water}}g$:

$$\frac{1}{\rho_{\text{air}}} = \frac{\rho_{\text{water}}\, g\, r^3}{\eta^2} \quad\Longrightarrow\quad r^3 = \frac{\eta^2}{\rho_{\text{air}}\,\rho_{\text{water}}\, g} = \frac{(1.8\times10^{-5})^2}{(1.2)(1000)(9.8)}\ \text{m}^3 \approx 2.8\times10^{-14}\ \text{m}^3.$$

So $r \approx 3\times10^{-5}$ m $= 30\ \mu$m. At that size the ratio from part (a) comes out to exactly 1, as it should. Weather scientists use a size close to this to separate cloud drops from drizzle.

(e) Why don't clouds fall?

They do fall, just very slowly. A cloud drop falls at about 1 cm/s, because for something that small, the stickiness of air controls the motion. Air inside a cloud rises at 10 cm/s to several m/s, which is 10 to several hundred times faster than the drops fall. So the drops get carried along with the air. Drops that do sink out the bottom of the cloud into drier air evaporate before they get far.

Extra challenge

Air rising at 1 m/s. It holds up any drop that falls slower than 1 m/s. Which formula? Try both.

Stickiness formula: $r = \sqrt{\eta v/(\rho_{\text{water}} g)} = \sqrt{(1.8 \times 10^{-5})(1)/9800}$ m ≈ 40 μm. But that's bigger than the 30 μm crossover, so this formula shouldn't be trusted here.

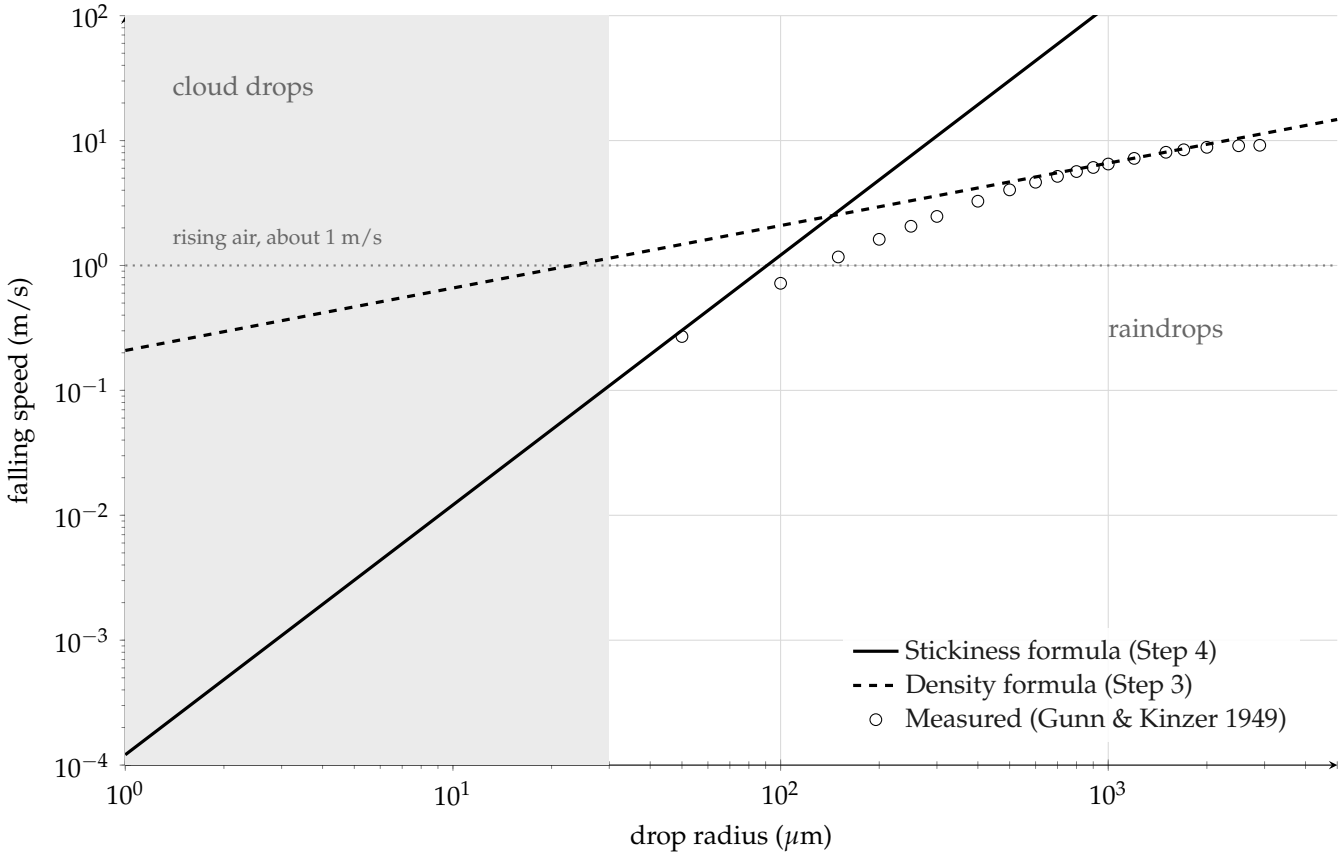
Density formula: $r = \rho_{\text{air}} v^2/(\rho_{\text{water}} g) = (1.2)(1)^2/9800$ m $\approx 1.2 \times 10^{-4}$ m ≈ 0.1 mm. Check the ratio: $(1.2)(1)(1.2 \times 10^{-4})/(1.8 \times 10^{-5}) \approx 8$, bigger than 1, so this formula is the better one.

Answer: drops smaller than about 0.1 mm in radius stay up. The measured value is close to this.

Evaporation. Big drops are more likely to reach the ground, for two reasons. They fall much faster, so they spend less time in the dry air. And a small drop has more surface for each bit of water inside it, so it loses its water faster. Small drops vanish on the way down; big ones make it.

THE WRAP-UP PLOT

Both formulas against real measurements



Both axes go up by factors of 10, so each formula shows up as a straight line. The solid line (stickiness) climbs steeply because v grows like r^2. The dashed line (density) climbs gently because v grows like $\sqrt{r}$. The real drops (circles) follow the stickiness line for small drops and the density line for raindrops, and bend in between. For the very biggest drops, the real speeds level off below the dashed line. That's because big drops flatten out and eventually break apart, which is the problem of the month.

WHOLE-ROOM DISCUSSION

Can we make it rain? Some ideas

These questions don't have a single right answer. Here are some of the main ideas.

1. A cloud full of 10 μm drops can hold a lot of water and just stay put. Each drop falls at about 1 cm/s, and the rising air holds it up.
2. The drops have to grow, about 100 times wider, which means about a million cloud drops joining together. One way: a drop that's a little bigger falls a little faster, runs into smaller drops, swallows them, and keeps growing.
3. Give the cloud a head start on making a few big particles. Utah's program releases tiny particles of silver iodide. In winter clouds, many drops stay liquid even below freezing; the particles give them something to freeze onto. The ice crystals then grow by taking water from the drops around them until they're heavy enough to fall as snow.
4. This is the hardest one. You can't run the same storm twice, once with seeding and once without. So you need something to compare against: nearby mountains that weren't seeded, many years of records from before and after, or choosing which storms to seed at random. Snowfall changes a lot from year to year on its own, so a small effect is hard to see.